UNIT - III

E. M. Waves in Matter

Propagation in Linear Media

Inside matter → in regions where → no free charge or free current → Maxwell's eq^n

$$\left.\begin{array}{ll}(i)\ \nabla\cdot D = 0 & (iii)\ \nabla\times E = -\dfrac{\partial B}{\partial t} \\ (ii)\ \nabla\cdot B = 0 & (iv)\ \nabla\times H = \dfrac{\partial D}{\partial t}\end{array}\right\} \quad \text{---(1)}$$

If medium is linear

$$D = \varepsilon E, \quad H = \frac{1}{\mu} B \quad \text{---(2)}$$

and homogeneous (so that ε and μ do not vary)

Maxwell's eq^{n}s

↳

$$\left.\begin{array}{ll}(i)\ \nabla\cdot E = 0 & (iii)\ \nabla\times E = -\dfrac{\partial B}{\partial t} \\ (ii)\ \nabla\cdot B = 0 & (iv)\ \nabla\times B = \mu\varepsilon\dfrac{\partial D}{\partial t}\end{array}\right\} \quad \text{---(3)}$$

vacuum analogs $\quad \mu_0\varepsilon_0 \longrightarrow \mu\varepsilon$ (replaced)

E. M. waves propagate through a linear homogeneous medium at a speed

$$v = \frac{1}{\sqrt{\varepsilon\mu}} = \frac{c}{n} \quad \text{---(4)}$$

where

$$n \equiv \sqrt{\frac{\varepsilon\mu}{\varepsilon_0\mu_0}} \quad \text{---(5)}$$

↓

index of refraction of the material.

Most of the materials $\mu \approx \mu_0$ so

$$n \cong \sqrt{\varepsilon_r} \quad \text{---(6)}$$

ε_r → dielectric constant

$\varepsilon_r \rightarrow$ always greater than 1.

Light travels more slowly through matter

So $\quad \varepsilon_0 \longrightarrow \varepsilon, \ \mu_0 \longrightarrow \mu$ and hence $c \longrightarrow v$

The energy density is

$$u = \frac{1}{2}\left(\varepsilon E^2 + \frac{1}{\mu} B^2\right) \quad \text{--- (7)}$$

and the Poynting vector

$$S = \frac{1}{\mu}(E \times B) \quad \text{--- (8)}$$

For monochromatic plane waves $\longrightarrow$ frequency and wave number are related by $\omega = kv$

amplitude B is $\frac{1}{v}$ times the amplitude of E and Intensity

$$I = \frac{1}{2} \varepsilon v E_0^2 \quad \text{--- (9)}$$

What happens $\longrightarrow$ a wave passes from one transparent medium $\longrightarrow$ to other.

Say air to water ?

glass to plastic ?

Boundary conditions for E-M. Waves

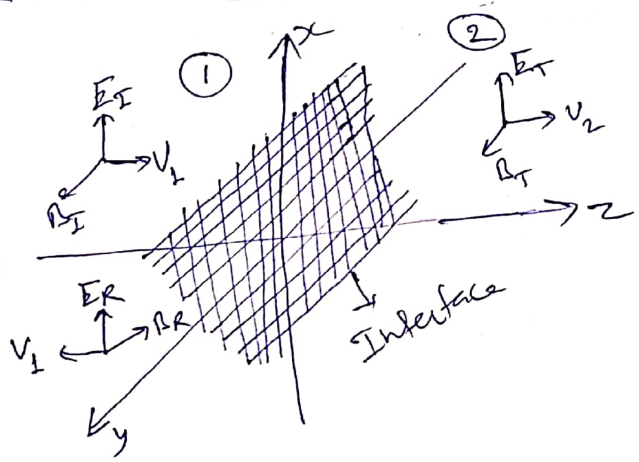
$$\left.\begin{array}{ll} \text{(i)} \ \varepsilon_1 E_1^{\perp} = \varepsilon_2 E_2^{\perp} & \text{(iii)} \ E_1^{\parallel} = E_2^{\parallel} \\ \text{(ii)} \ B_1^{\perp} = B_2^{\perp} & \text{(iv)} \ \frac{1}{\mu_1} B_1^{\parallel} = \frac{1}{\mu_2} B_2^{\parallel} \end{array}\right\} \text{--- (10)}$$

These eq's relate the electric and magnetic fields just to the left and just to the right of the interface between two linear media.

Laws of governing reflection and refraction of E-M. Waves $\longrightarrow$?

Reflection and Transmission at Normal Incidence

xy plane → boundary between two linear media



A plane wave of frequency ω

→ traveling in the z-direction and polarized in the x-direction

↳ approaches the interface from the left.

$$\left.\begin{aligned} \tilde{E}_I(z,t) &= \tilde{E}_{0I}\, e^{i(k_1 z-\omega t)}\,\hat{x} \\ \tilde{B}_I(z,t) &= \frac{1}{v_1}\tilde{E}_{0I}\, e^{i(k_1 z-\omega t)}\,\hat{y} \end{aligned}\right\} \quad (11)$$

It gives rise to a reflected wave

$$\left.\begin{aligned} \tilde{E}_R(z,t) &= \tilde{E}_{0R}\, e^{i(-k_1 z-\omega t)}\,\hat{x} \\ \tilde{B}_R(z,t) &= -\frac{1}{v_1}\tilde{E}_{0R}\, e^{i(-k_1 z-\omega t)}\,\hat{y} \end{aligned}\right\} \quad (12)$$

Travels back to the left in medium 1

and a transmitted wave

$$\left.\begin{aligned} \tilde{E}_T(z,t) &= \tilde{E}_{0T}\, e^{i(k_2 z-\omega t)}\,\hat{x} \\ \tilde{B}_T(z,t) &= \frac{1}{v_2}\tilde{E}_{0T}\, e^{i(k_2 z-\omega t)}\,\hat{y} \end{aligned}\right\} \quad (13)$$

↓

Continues on the right in medium (2)

Minus sign in $\tilde{B}_R$ → so that Poynting vector aims in the direction of propagation.

At $z=0$ in accordance with boundary conditions (iii), (iv) of eq^n (10)

fields on left must join fields on right

$\tilde{E}_I + \tilde{E}_R$ and $\tilde{B}_I + \tilde{B}_R$ $\qquad$ $\tilde{E}_T$ and $\tilde{B}_T$

There are no components $\rightarrow$ perpendicular to the surface so (i) and (ii) are trivial of eq^n (10)

(iii) requires $\leftarrow$ $\tilde{E}_{0I} + \tilde{E}_{0R} = \tilde{E}_{0T}$ --- (14)

(iv) $\rightarrow$ $\frac{1}{\mu_1}\left(\frac{1}{v_1}\tilde{E}_{0I} - \frac{1}{v_1}\tilde{E}_{0R}\right) = \frac{1}{\mu_2}\left(\frac{1}{v_2}\tilde{E}_{0T}\right)$ --- (15)

or $\quad \tilde{E}_{0I} - \tilde{E}_{0R} = \beta\,\tilde{E}_{0T}$ --- (16)

where $\quad \beta \equiv \frac{\mu_1 v_1}{\mu_2 v_2} = \frac{\mu_1 n_2}{\mu_2 n_1}$ --- (17)

Solving eq^n's (14) and (16) to obtain outgoing amplitudes $\rightarrow$ in terms of the incident amplitude

$$\tilde{E}_{0R} = \left(\frac{1-\beta}{1+\beta}\right)\tilde{E}_{0I}\,, \quad \tilde{E}_{0T} = \left(\frac{2}{1+\beta}\right)\tilde{E}_{0I} \quad \text{--- (18)}$$

Similar results as $\rightarrow$ for waves on a string.

If permeabilities μ $\rightarrow$ close to their values in vacuum, then $\beta = \frac{v_1}{v_2}$ and we have

$$\tilde{E}_{0R} = \left(\frac{v_2 - v_1}{v_2 + v_1}\right)\tilde{E}_{0I}\,, \quad \tilde{E}_{0T} = \left(\frac{2 v_2}{v_2 + v_1}\right)\tilde{E}_{0I}$$

--- (19)

Reflected wave → in phase if $v_2 > v_1$
and out of phase if $v_2 < v_1$; → real amplitudes are related by

$$E_{0R} = \left|\frac{v_2 - v_1}{v_2 + v_1}\right| E_{0I} \quad , \quad E_{0T} = \left(\frac{2 v_2}{v_2 + v_1}\right) E_{0I} \quad -(20)$$

or, in terms of indices of refraction

$$E_{0R} = \left|\frac{n_1 - n_2}{n_1 + n_2}\right| E_{0I} \quad , \quad E_{0T} = \left(\frac{2 n_1}{n_1 + n_2}\right) E_{0I} \quad -(21)$$

fraction of incident energy reflected ?
fraction of incident energy transmitted ?

The intensity (average power per unit area) is

$$I = \frac{1}{2} \varepsilon v E_0^2$$

if $\mu_1 = \mu_2 = \mu_0$, ratio of the reflected intensity to the incident intensity is

Reflection Coefficient ← $$R \equiv \frac{I_R}{I_I} = \left(\frac{E_{0R}}{E_{0I}}\right)^2 = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2 \quad -(22)$$

Ratio of the transmitted intensity to the incident intensity is

Transmission Coefficient ← $$T \equiv \frac{I_T}{I_I} = \frac{\varepsilon_2 v_2}{\varepsilon_1 v_1}\left(\frac{E_{0T}}{E_{0I}}\right)^2 = \frac{4 n_1 n_2}{(n_1 + n_2)^2} \quad -(23)$$

$$\Rightarrow \boxed{R + T = 1} \quad -(24)$$

Ex: When light passes from air ($n_1 = 1$) ~~into~~ into glass ($n_2 = 1.5$)
$R = 0.04$ and $T = 0.96$ → Most of the light transmitted